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Turnout Across Democracies

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World democracies widely differ in legislative, executive, and legal institutions. Different institutional environments induce different mappings from electoral outcomes to the distribution of power. We explore how these mappings affect voters' participation in an election. We show that the effect of such institutional differences on turnout depends on the distribution of voters' preferences. We uncover a novel contest effect: Given the preferences distribution, turnout increases and then decreases when we move from a more proportional to a less proportional power-sharing system; turnout is maximized for an intermediate degree of power sharing. Moreover, we generalize the competition effect, common to models of endogenous turnout: Given the institutional environment, turnout increases in the ex ante preferences evenness, and more so when the overall system has lower power sharing. These results are robust to a wide range of modeling approaches, including ethical voter models, voter mobilization models, and rational voter models.

Electoral participation considered as an indication of the health of a democracy (Powell 1982; Rose 1980) and a pillar of the democratic ideal of political equality (Lijphart 1997). The historical average turnout rate as a percentage of voting age population displays a large variance across world democracies: New Zealand, Portugal, Indonesia, Italy, and Albania have an average turnout rate above 85%, whereas Senegal, Colombia, Ecuador, Ghana, and the United States all have average turnout rates below 50%.[1] A number of empirical papers have attempted to account for cross-national variations in turnout (Black 1991; Blais and Carty 1990; Blais and Dobrzynska 1998; Crewe 1981; Franklin 1996; Jackman 1987; Jackman and Miller 1995; Powell 1980, 1982, 1986; Selb 2009). These studies highlight that electoral rules and party systems are important factors affecting turnout but, by and large, do not focus on other political institutions and, more importantly, neglect their interaction in determining the influence votes have on policy outcomes.

Voters do not care about the distribution of seats per se, but rather about the overall power to change policies conferred by those seats. Any theory of turnout should focus on the incentives of voters and, thus, should consider the mapping from how seats are distributed to how power is shared, not only the mapping from votes to seats. This raises an important question: How do political institutions—that is, not only electoral rules, but also forms of government, bicameralism, judicial review, federalism, separation of powers, committee chair assignments, and all other institutions determining the power of the majority and minorities for any given distribution of seats in a legislature—affect electoral participation in a democracy? This article aims to provide a set of robust theoretical predictions about how turnout varies with the overall proportionality of the mapping from the distribution of votes to the distribution of power, hence skipping completely the intermediate step of the distribution of seats.

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[1]Pintor, Gratschew, and Sullivan (2002) report these figures, based on electoral data from all parliamentary elections since 1945.

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A recent theoretical literature in economics and political science has compared turnout in two extreme cases of power sharing: a winner-take-all benchmark where the winner of the majority of votes receives 100% of the power to decide on policies, and the opposite extreme benchmark of full proportionality of the mapping from votes to power shares (Faravelli and Sanchez-Pages Forthcoming; Herrera, Morelli, and Palfrey 2014; Kartal, Forthcoming a). This is unsatisfactory, as most institutional systems in use around the world are de jure or de facto rather somewhere in between those two extremes. Not only do many countries adopt explicitly mixed electoral systems, where two electoral rules using different formulae run alongside each other,[2] but also formal and informal institutions in the legislative, executive, or judicial branches induce different mappings from electoral outcomes to the distribution of power. Examples of such institutions include the veto power of a qualified minority in the United States, the way margins of victory translate into committee assignments or guarantee a more powerful mandate, the division of powers between the legislature, and the executive, federalism, and the power to appoint constitutional judges.

In this article, we develop a formal approach in order to provide rigorous foundations for the study of the complex relationship between the proportionality of an overall institutional system and turnout. We present a theoretical analysis of the fundamental causes of the variation in turnout based on differences in institutions for political power sharing. Instead of exploring all the formal and informal political institutions mentioned above, we consider all possible determinants of the degree of power sharing in a reduced form, considering as equally important all the different institutional components affecting the mapping from vote shares to the relative weight of different parties in policymaking (henceforth *power shares*). We use the "contest success function"[3] and introduce a power-sharing parameter, γ, that allows us to embed a wide array of institutional systems or power-sharing regimes, ranging from a fully proportional power-sharing system ($\gamma = 1$) to a system with zero power sharing ($\gamma = \infty$). This modeling innovation allows us to span continuously across all institutional systems and to analyze them. We study the role of these institutions in electoral participation by characterizing how the vote-shares-to-power-shares mapping affects voters' incentives to vote and parties' campaign efforts. We try to develop a theory that is as robust and general as possible. First, we take into account the distribution of political preferences in the population, a contextual factor that has proven to be crucial in models of endogenous turnout (see, e.g., Herrera, Morelli, and Palfrey 2014; Kartal, Forthcoming a). Second, we allow for multiple alternative behavioral assumptions about the turnout mechanics, rather than limiting our analysis to one single approach.

We show that the effect of the institutional differences on turnout depends on the distribution of voters' preferences for the competing parties, or the ex ante preference "evenness" of the election in a nonobvious way.[4] In particular, we uncover a novel *contest effect*: Given any asymmetric distribution of preferences, as we move gradually from a relatively even power-sharing system to one that gives more policymaking power to the majority party, turnout increases first and then decreases. Therefore, turnout is highest for intermediate levels of the overall institutional mapping from votes to power. The turnout-maximizing degree of power sharing depends on the distribution of preferences, but it is always intermediate for any ex ante uneven election. The intuition is as follows. As we move away from an even power-sharing system, the institutional system becomes more and more similar to a system where power is concentrated in the hands of the party that obtains a plurality of the votes. Hence, turnout drops for any lopsided preference distribution because the underdog side has no chance of obtaining the plurality of the votes, which is all that matters in this case. A system with even power sharing will typically display moderate turnout for all preference distributions, as the incentives to turnout remain even in a very lopsided election: There is always a possible power gain for turning out more. Finally and crucially, for intermediate systems—that is, between full power sharing and no power sharing—turnout is the highest. To understand the intuition for this result, imagine that the preference split is 40-60 and think about which institutional system would grant the largest turnout in this case. Intuitively, such a system would be one that grants significant power gains around an election outcome close to 40-60: namely, an intermediate system whose vote-shares-to-power-shares mapping is very steep around a qualified minority of 40%, so that the marginal

[2]See, for example, Moser and Scheiner (2004) for a comparative analysis of mixed electoral systems.

[3]See Tullock (1980). This function is extensively used in several economic contexts, especially in the contest literature (see, among many others, Skaperdas 1996) typically as a mapping from efforts or resources to the chance of victory.

[4]Cox (1999, p.393), in summary of the analysis of the elite mobilization section, says that "the argument following Key (1949) says that closeness will (a) boost mobilizational effort and (b) correlate positively with turnout." Our model will qualify these statements for each degree of power sharing and hence comparatively. We do this not only for the mobilization logic but also from the instrumental and ethical voting perspectives.

gain from extra votes can make a significant difference in the powers granted de jure or de facto.

In addition to this, we generalize to all institutional environments the *competition effect*, already well documented in several models of endogenous turnout: Given the institutional environment, turnout increases in the ex ante preference evenness of the election and peaks when the population is perfectly evenly split between the two parties.[5] Even though such a competition effect is common to all institutional systems, the sensitivity of turnout to the level of competitiveness is higher the lower is the extent of power sharing, at least when excluding situations with especially large expected margins of victory.

We derive our results for the ethical voter model and then show that these results are preserved in other costly voting models. These models are the voter mobilization model, which we fully characterize, and the rational voter model, for which we provide numerical simulations supporting the qualitative results of the previous two models. Unlike the rational voter model, the ethical voter model, for which we conduct the core of our analysis, assumes that voters on the two sides overcome the free-rider problem so that each side turns out at the optimal level. This guarantees that turnout remains large in a large election, a desirable property. On the technical side, the decreasing generalized reversed hazard rate (DGRHR) property of the cost function, a regularity condition on probability distributions, turns out to be the key sufficient condition to obtain all the results in all group models. The equilibria from all models feature another well-documented property (see, among others, Castanheira 2003), the *underdog effect*. We show that, in all the models we present and for all institutional systems, the underdog effect is *nonfull*, which means that the side enjoying the majority of ex ante support also obtains the majority of the votes in equilibrium for all power-sharing systems. This property drives all comparative statics results we obtain, namely, the competition effect and, most importantly, the contest effect described above.

Related Literature

Our modeling strategy is related to a body of literature that studies voters' turnout in large elections. Our main model is the ethical voting model (Coate and Conlin 2004; Feddersen and Sandroni 2006). We also show the same results hold for mobilization models (Cox and Munger 1989; Morton 1987, 1991; Shachar and Nalebuff 1999; Uhlaner 1989). Razin (2003) studies the effect of vote shares on policy platforms. He shows that vote shares communicate information to the candidates, who consequently have an incentive to moderate their policy when their margin of victory shrinks. Castanheira (2003) is, to our knowledge, the first article to consider the effect of "mandates" on turnout in large elections. Its focus is not on comparing the size of mandates per se, but on showing that mandates have in general the effect of dramatically increasing turnout relative to political systems without a mandate effect.

A recent strand of literature—including, among others, Herrera, Morelli, and Palfrey (2014) and Kartal (Forthcoming a, b)—studies strategic voting in a rational voter framework and analyzes how turnout varies in two extreme electoral systems (i.e., fully proportional power sharing and no power sharing) for all levels of preference splits. These article cannot say much about intermediate electoral systems or, more importantly, about the overall level of power sharing of an institutional system. Kartal (Forthcoming a) focuses more on comparative welfare results than on comparing turnouts. As far as turnout is concerned, Kartal (Forthcoming a) (see also Herrera, Morelli, and Palfrey 2014) shows that full underdog compensation, and hence a close high-turnout election, can occur when the distribution of voting costs is degenerate (see also Goeree and Grosser 2007; Taylor and Yildirim 2010), or is bounded below by a strictly positive minimum voting cost (see also Krasa and Polborn 2009) but not otherwise. Faravelli and Sanchez-Pages (Forthcoming) is the first article comparing turnout and welfare across a wider range of power-sharing rules, in the same spirit as we do here. They obtain results in a neighborhood of elections with preferences that are ex ante perfectly even or perfectly biased, the only tractable cases in a rational voter model. Ours is the first article that studies a continuum of institutional systems for a general distribution of preferences in the population. Faravelli, Man, and Walsch (2013) study the effect of mandate together with "paternalism." They show, under very general conditions, that the combination of these two factors guarantees positive turnout even in a large elections and in a rational voter framework. In addition, they provide evidence of a mandate effect from U.S. congressional elections.

Finally, the article speaks to the empirical literature that studies cross-national variations in turnout (Black 1991; Blais and Carty 1990; Blais and Dobrzynska 1998; Crewe 1981; Franklin 1996; Jackman 1987; Jackman and Miller 1995; Powell 1980, 1982, 1986; Selb 2009). The theoretical results we obtain from all models, from instrumental voting to mobilization models, depend on a key variable, namely, the expected winning margin or the

[5]Several experimental works have confirmed this theoretical prediction (see Herrera, Morelli, and Palfrey 2014; Kartal, Forthcoming b; Levine and Palfrey 2007).

closeness of the election. While there is some empirical evidence about the relationship between ex ante closeness and turnout (Blais 2000; Cox and Munger 1989; Selb 2009), the interaction effect of the expected closeness and the degree of power sharing of the institutional system has not been studied in the way we propose, that is, taking into account the important mapping from the distribution of seats to the distribution of power. The article is theoretical, but, in the Online Supporting Information, we provide indications and examples about how future empirical research could properly take into account the message and methodological points of this article. In particular, if what matters for voters' participation is the overall mapping from votes to power (not the intermediate mapping from votes to seats), a theoretical prediction on how turnout depends on the proportionality of the whole political system can be tested using the proportionality indices for the electoral rules only across countries with similar mappings from seats to power, such as with similar division of power between the legislature and the executive.

The article is organized as follows. The next section presents the general model setup. The third section contains the analysis of group voting models—that is, the ethical voter model and the mobilization voter model — and compares turnout across different institutional environments and preferences distributions. The fourth section shows that similar results hold if we consider, instead, a rational voter framework. The last section offers some concluding remarks and describes potential paths of future research.

General Setup

We introduce here a setup common to all models we consider. Consider two parties, A and B, competing for power.[6] Citizens have strict political preferences for one or the other. We denote by $q \in (0, 1)$ the preference split, that is, the chance that any citizen is assigned (by Nature) a preference for party A (thus, $1 - q$ is the expected fraction of citizens who prefer party B). Besides partisan preferences, the second dimension along which citizens differ from one another is their cost of voting: Each citizen's cost of voting, c, is drawn from a distribution with twice differentiable cumulative distribution function $F(c)$ over the support $c \in [0, \bar{c}]$, with $\bar{c} > 0$. The cost of voting and the partisan preferences are two independent dimensions that determine the type of voter.

For any vote share V obtained by party A, an institutional system γ determines the mapping to the respective power shares, $P_\gamma^A(V) \in [0, 1]$ and $P_\gamma^B(V) = 1 - P_\gamma^A(V)$. For normalization purposes, we let the utility from "full power to party i" equal 1 for type i citizens and 0 for the remaining citizens.[7]. Hence, the power shares are the reduced-form "benefit" components of parties' (respectively, voters') utility functions that will determine the incentives to campaign (respectively, vote) in a given institutional system γ. In a $\gamma-$system, payoffs as a function of the vote share are represented by a standard "contest success function,"[8] where γ ranges from 1 to ∞:

$$P_\gamma^A(V) = \frac{V^\gamma}{V^\gamma + (1 - V)^\gamma},\ P_\gamma^B(V) = \frac{(1 - V)^\gamma}{V^\gamma + (1 - V)^\gamma}. \quad (1)$$

This representation can accommodate a wide range of intermediate power-sharing rules between pure proportional power-sharing systems (P) and systems entirely without power sharing (M), using a single parameter in the payoff function. The two extreme cases correspond to $\gamma = 1$ (P) and $\gamma = \infty$ (M), and, for instance, the intermediate case $\gamma = 3$ represents the so-called "cube law."[9]

As we discussed in the introduction, intermediate systems that are a mixture of proportional power-sharing and no-power-sharing systems are very common and have plenty of institutional details we do not model.[10] Intuitively, we just want to capture the fact that the larger γ is, the lower the extent of power sharing in the system.

Even in a winner-take-all electoral system like the U.S. presidential race, a large winning margin carries with it added benefits to the winner due to a "mandate" effect, and larger winning margins for the president can carry over to a larger majority in one or both houses of

[6] We assume two parties with fixed platforms. It would be interesting to study voters' turnout decisions when the level of power sharing of the systems also affects political platforms and the endogenous entry of parties. For a recent article linking electoral rule disproportionality to platform polarization, see Matakos, Troumpounis, and Xefteris (2014).

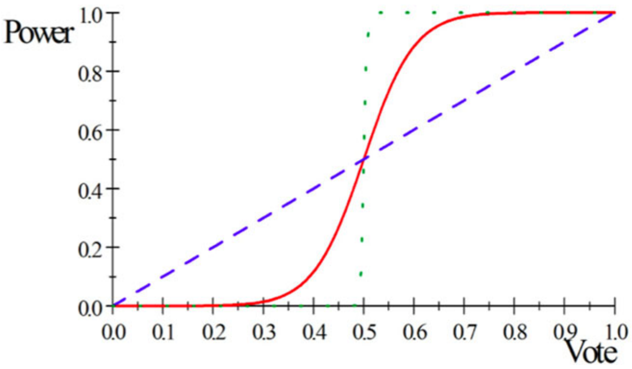
[7] This normalization will allow us to match party utility and voters' utilities in a simple way under all the models that will be considered.

[8] See, for instance, Hirshleifer (1989). When nobody votes ($\alpha = \beta = 0$), assume equal shares ($V = 1/2$).

[9] There are other ways to introduce a power-sharing-level parameter. Faravelli and Sanchez-Pages (Forthcoming) model it as a linear combination of the payoffs in two systems, P and M. For a recent article linking electoral rule disproportionality to platform polarization, see Matakos, Troumpounis, and Xefteris (2014).

[10] For example, in Germany, voters express two preferences, one for a candidate and one for a party: 299 members of parliament's lower house are directly elected in single-member districts; another 299 members are elected from candidate lists until each party's seat share matches the proportion of party votes that it won.

FIGURE 1 Power-Sharing Function for Different Values of γ



Congress, via the "coattails" effect.[11] Also, the fact that the legislative branch in an M system has leverage over the executive branch and the presidency will tend to smooth out the winner-take-all discontinuous payoff function in the direction of a more proportional power-sharing scheme. Similarly, an increase in vote shares might have a disproportional impact on payoffs also in electoral systems with proportional representation. For example, in parliamentary systems that require the formation of a coalition government, a party that is fortunate to win a clear majority of seats outright has much less incentive (or in some cases none at all) to compromise with other parties in order to govern effectively.

Figure 1 illustrates the power share payoff P_γ^A as a function of the vote share V for three power-sharing parameters γ, namely: $\gamma = 1$ (i.e., the P system, dashed line), $\gamma = 5$ (i.e., an intermediate power-sharing system, continuous line), and $\gamma \to \infty$ (i.e., a pure M system, dotted line).

Citizens choose whether to vote for party A, vote for party B, or abstain. If a share α of A types vote for A and a share β of B types vote for B, the expected turnout for party A and party B and total turnout are, respectively:

$$T_A := q\alpha, \quad T_B := (1-q)\beta, \quad T := T_A + T_B,$$

whereas the expected vote shares for party A and party B are, respectively:

$$V = \frac{q\alpha}{T}, \quad 1 - V = \frac{(1-q)\,\beta}{T}.$$

Without loss of generality, in the remainder we assume party A is the ex ante *underdog*, namely, $q \in [0, 1/2]$ where applicable . We also define the preference ratio Q and the turnout ratio R as

$$Q := \frac{q}{1-q}, \quad R := \frac{T_A}{T_B}.$$

We look for symmetric equilibria. These equilibria can be characterized by a voting cost threshold for each side (c_α, c_β), below which supporters turn out and above which they abstain; hence, the share of A (B) supporters who turn out can be expressed by $\alpha = F(c_\alpha)$ $(\beta = F(c_\beta))$. Henceforth, we denote as $f(c) = F'(c)$ the probability density function of the cumulative cost distribution function $F(c)$, and we call G its inverse, namely, $G(\alpha) := F^{-1}(\alpha) = c_\alpha$. Moreover, we denote partial derivatives of any function Z with respect to q or γ (our main comparative statics parameters) with the following compact notation: $Z_q := \frac{\partial Z}{\partial q}$.

Group Voting Models

The basic idea behind these models is that the positive externality of voting among supporters of the same party is internalized, leading to higher turnout. The rationale behind the solution to this collective action problem may differ across group voting models, but the end result is that, contrary to the instrumental voting model (discussed in the next main section), the share of voters turning out is high regardless of the size of the population. In group voter models, the two sides compete in an election by turning out their supporters, who have a voting cost to turn out. The population is a continuum of measure one, divided into q A supporters and $(1-q)$ B supporters. In a $\gamma-$ power-sharing system, the marginal group benefits to the two sides, with respect to (c_α, c_β), can be derived from Equation (1) and are, respectively,

$$\frac{dP_\gamma^A}{dc_\alpha} = \frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\,\beta}{T^2}\right) q f(c_\alpha),$$
$$\frac{dP_\gamma^B}{dc_\beta} = -\frac{dP_\gamma^A}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\, f(c_\beta), \qquad (2)$$

where

$$\frac{dP_\gamma^A}{dV} = -\frac{dP_\gamma^B}{dV} = \frac{\gamma}{V(1-V)} \frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1 + \left(\frac{V}{1-V}\right)^\gamma\right]^2}.$$

Ethical Voter Model

Our main approach to studying turnout in elections, which is grounded in group-oriented behavior, is the ethical voter model (Coate and Conlin 2004; Feddersen and Sandroni 2006). This model assumes that citizens are rule

[11]For an empirical analysis of such effects, see Ferejohn and Calvert (1984) and Calvert and Ferejohn (1983). See also Golder (2006).

utilitarian, which means that they overcome the free-riding problem and manage to act as one cohesive group. Since the ethical voter model assumes that citizens are rule utilitarian, all citizens of one group act as one agent. Namely, we assume citizens follow the voting rule that, if followed by everyone else on their side, would maximize the benefit P_γ^j of their side from the outcome of the election minus the aggregate cost C incurred by their side. As a consequence, this model involves an equilibrium between two party planners, or representative agents, on each side, A and B. In the solution, each planner looks at the total electoral benefit net of the total cost of voting incurred by his supporters, taking the other planner's turnout strategy as given.[12] The cost of turning out the voters for the social planner on side A is the total cost suffered by all the citizens on side A who vote, namely,

$$C(c_\alpha) := q\int_0^{c_\alpha} c f(c)\, dc.$$

Each side's optimal voting rule specifies a critical cost level below which an individual should vote. The citizens with cost below the planner-chosen cost threshold, c_α, vote because ethical voter models assume citizens get an exogenous benefit D (larger than their private voting cost $c \leq c_\alpha$) for "doing their part" in following the optimal rule established by the planner.

Defining the *generalized reversed hazard rate* as $\frac{cf(c)}{F(c)}$, we introduce the following definition: A distribution satisfies the *decreasing generalized reversed hazard rate* (DGRHR) property if and only if $\frac{cf(c)}{F(c)}$ is decreasing. We call it DGRHR by analogy with the known increasing generalized failure rate (IGFR; see, e.g., Lariviere 2006), which refers to the function $\frac{cf}{1-F}$.[13]

To insure corner solutions are ruled out, it suffices to assume a cost density function that has two additional boundary conditions. First, a large enough density for low costs: this rules out the zero turnout best response by making it almost costless to turn out the first voters. Second, a large enough support: this makes it unfeasible to turn out all voters.[14] Namely:

Condition 1. *The cdf of the cost distribution $F(c)$ defined on $c \in [0, \overline{c}]$ must satisfy the additional boundary conditions:*

1) $$\lim_{c\longrightarrow 0} \frac{F(c)}{c^{1/k}} > 0 \quad \text{for } k > \gamma - 1$$

2) $$\overline{c} > \frac{\gamma}{4q}$$

Thanks to the assumed DGRHR property, the boundary conditions above not only rule out corner solutions but also guarantee that the objectives of both A and B are single peaked. Thus, we have the following result for all $q \in [0, 1/2]$

Proposition 1. *The equilibrium exists, and it is unique and has the following properties:*

(1) Partial Underdog Compensation: For $q < 1/2$, we have $\alpha > \beta$, $q\alpha < (1-q)\beta$, namely, underdog supporters turn out at a higher rate than leader supporters, $R < 1$.
(2) Competition Effect: Given an institutional system γ, turnout, T, and turnout ratio, R, increase in the evenness of the preference split, q.
(3) Contest Effect: Given the preference split, q, turnout increases and then decreases with the unevenness of the power sharing γ; it achieves its maximum for intermediate γ.
(4) As γ goes to infinity (no power sharing), turnout goes to one when the election is ex ante even, $q = 1/2$, and goes to zero otherwise.

Proof. See the appendix. ■

The solution for the ethical voter model for a general γ is not straightforward to derive, because the underdog compensation is strictly partial (rather than zero), so $\alpha \neq \beta$, and the two equations of the system of first order conditions (FOCs) do not decouple. It is convenient for the analysis to rewrite the two FOCs compactly as

$$W = q\alpha G(\alpha) = (1-q)\beta G(\beta),$$

where:

$$W := \gamma \frac{R^\gamma}{[1+R^\gamma]^2}.$$

The DGRHR property turns out to be key for several reasons, not only to guarantee existence, but also for the competition effect and for the contest effect. It is easy to show nonexistence if DGRHR is violated, at least in certain parameter ranges. Even when existence is granted, a violation of DGRHR can cause the competition effect to fail, that is, higher equilibrium turnout in more lopsided elections, as we show later in an example. The DGRHR property guarantees some regularity in the cost distribution function, so that, for instance, if the ratio of the

[12]We assume "collectivism," so the planner on each side, A and B, only looks at the total cost of voting of the voters on his side. The results would not change if we assumed "altruism," as in Feddersen and Sandroni (2006): Each planner takes into account the cost of voting of all citizens who vote regardless of their side.

[13]The DGRHR is also a key regularity condition in strategic models of turnout such as Faravelli, Man, and Walsh (2013). Che, Dessein, and Kartik (2013) in their Appendix G verify that a variety of familiar classes of distributions, including Pareto distributions, power function distributions (which subsume uniform distributions), Weibull distributions (which subsume exponential distributions), and gamma distributions, satisfy this condition.

[14]We thank David Levine for drawing our attention to the fact that the standard first order conditions might not be sufficient, especially for large γ. He was right.

proportion of voters turning out from each side—α/β—increases as parameters change, then the cost threshold ratio—c_α/c_β—also increases. The latter implies, among other things, that there is monotonicity between the relative support ex ante and the relative support ex post: If q, the relative ex ante support for A increases, then the relative turnout for A, $R = T_A/T_B$, does too in equilibrium. For instance, if 1 out of 4 citizens prefers A and 1 out of 3 voters actually voted for A (see the partial underdog effect described below), then it cannot be that increasing the former reduces the latter, under DGRHR. We now discuss the properties we derived in order.

(1) What seems to be a common feature across several costly voting models is the underdog effect (see Castanheira 2003 and Levine and Palfrey 2007, among others). Namely, voting models have the general property that the supporters for the underdog side have higher incentive to turn out than the leader's supporters. Hence, in equilibrium they vote disproportionally more than the leader's supporters. We call this feature "compensation," as it reduces the leader's initial advantage. Moreover, we call this compensation "partial" when the larger turnout of the underdog supporters is not enough to overturn the leader's initial advantage in preferences. In rational voter models, partial compensation seems to be a general feature when voting costs are heterogeneous (see also Herrera, Morelli, and Palfrey 2014; Kartal Forthcoming a), the reason being that while the underdog side has higher benefits from turning out, it also bears higher costs, as it needs to turn out more supporters, and hence will only find more supporters with higher costs. In the ethical model analyzed here, though, the compensation effect comes from the cost side and not from the benefit side, as in rational voter models. More specifically, the benefit side is in fact identical for the two competing sides. First, because it is a zero sum game, there is a symmetry between the incentives of one side and the incentives of the other. Second, the left and right derivatives are identical in a continuous model. This means that, from any starting turnout profile, marginally increasing the turnout of one side increases the benefit to that side as much as marginally increasing turnout for the opposite side increases the benefit to the opposite side. Hence, the difference lies on the cost sides alone, namely, for any given cost threshold c_α, the underdog party has lower additional cost to turn out those additional voters. In other words, the marginal cost is

$$q c_\alpha f(c_\alpha),$$

and it is lower for the underdog ($q < 1/2$). This compensation, however, is partial, so the ex ante leader remains the ex post leader, albeit by a lower margin. This happens because of the term c_α in the marginal cost, which means that it costs more to turn out additional agents. This is the same logic described above for the rational voter model.

(2) We show that the competition effect is general for the ethical voter model regardless of the electoral or institutional system: the closer the ex ante preference split, the larger the turnout. While the competition effect seems like a very intuitive property, it is in fact not generally true in rational voter models, even with a system without power sharing.

(3) We believe the contest effect is novel—previous work has not compared turnout across different institutional systems for all preference splits q (for extreme values of q, see Faravelli and Sanchez-Pages (Forthcoming)). The intuition is as follows: Take any value of the preference split, say, for instance, a 40-60 preference split ($q = 40\%$). When γ is large, the system becomes similar to a system without power sharing. Hence, turnout should be low because, despite the underdog compensation, the underdog side has a very small chance of winning. When γ is low, the system becomes similar to a proportional power-sharing system, with moderate turnout for all preference splits. For intermediate γ, the marginal gain from extra votes can make the most difference, and this is where turnout is highest. For instance, an intermediate γ could model in reduced form an electoral system (and possibly several other institutional details) where extra votes for the underdog around a 40-60 outcome could mean leadership in more committees and/or obtaining veto or filibuster powers for some decisions. Having a partial (rather than full) underdog effect is crucial for the contest effect: An election that is not a toss-up ex ante needs to remain such ex post. If, in equilibrium, we had a 50-50 electoral outcome, then turnout would always be increasing in γ. This is because the slope of the power function is steepest around a 50-50 outcome in all institutional systems (for all $\gamma > 1$).

(4) Lastly, in a system with no power sharing, turnout is high and positive only in a very close election, because due to the *partial* underdog compensation, the underdog side has no chance of winning an ex ante lopsided election. Hence, the underdog supporters will not turn out significantly in such an election.

Examples. We provide first a simple example that satisfies DGRHR. Second, to show that DGRHR is a tight condition, we provide an example that violates it and violates the competition effect.

Closed-Form Example. Assume the cdf comes from the family (which satisfies weakly DGRHR):

$$F(c) = \left(\frac{c}{\bar{c}}\right)^{1/k}, c \in [0, \bar{c}], \quad k \geq 1 \Longleftrightarrow G(\alpha) = \bar{c}\alpha.^{k}.$$

with (to rule out corner solutions):

$$k > \gamma - 1, \qquad \bar{c} > \frac{\gamma}{4q}$$

The FOCs are

$$q\bar{c}\alpha^{1+k} = (1-q)\,\bar{c}\beta^{1+k} = W,$$

where

$$R = Q^{\frac{k}{1+k}},\ W = \left(\gamma \frac{Q^{\gamma \frac{k}{1+k}}}{\left(1 + Q^{\gamma \frac{k}{1+k}}\right)^2}\right)$$

Hence, turnouts for each side are

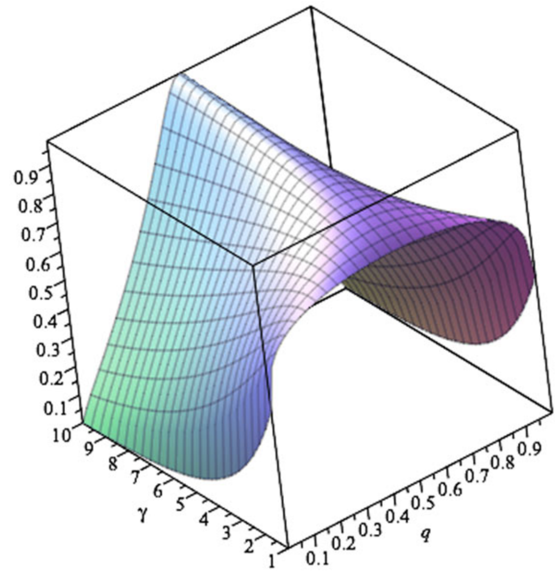
$$\alpha = \left(\frac{\gamma}{q\bar{c}} \frac{Q^{\gamma \frac{k}{1+k}}}{\left(1 + Q^{\gamma \frac{k}{1+k}}\right)^2}\right)^{\frac{1}{1+k}},$$

$$\beta = \left(\frac{\gamma}{(1-q)\,\bar{c}} \frac{Q^{\gamma \frac{k}{1+k}}}{\left(1 + Q^{\gamma \frac{k}{1+k}}\right)^2}\right)^{\frac{1}{1+k}}$$

Figure 2 shows T as a function of both q and γ for or $k = 10$ and $\bar{c} = 6$.

This picture summarizes the main insights. For any electoral/institutional system γ, the competition effect is apparent: Turnout increases the closer the ex ante preference split becomes. Fixing the preference split q, turnout is nonmonotonic in the electoral/institutional system γ, first increasing and then decreasing. For any $q < 1/2$, the turnout maximizing $\widehat{\gamma}$ is increasing in the competition q: The closer the election, the more uneven the power sharing in the institutional system has to be in order to achieve its highest turnout. In other words, if ex ante preference splits are uneven, then more proportional power-sharing systems maximize turnout; on the other hand, if ex ante preference splits are even, systems with less power sharing achieve the highest turnout. In sum, the electoral/institutional system that delivers the highest turnout crucially depends on the initial preference split and in a nontrivial way. This questions the validity of all cross-country empirical comparisons of turnout, which, to the best of our knowledge, lump together electoral turnout results over time in each country, never controlling for the value of q in each election.[15]

Figure 2 T as a Function of γ and q, Ethical Voter Model



Counterexample. In general, from the FOCs we obtain turnout

$$T = \gamma\,(G(\alpha)\,G(\beta))^{\gamma-1} \frac{G(\alpha) + G(\beta)}{((G(\alpha))^{\gamma} + (G(\beta))^{\gamma})^2}.$$

For $\gamma = 1$, we have

$$T = \frac{1}{G(\alpha) + G(\beta)}.$$

The cdf family

$$F(c) = 1 - (1-c)^{1/m} \Longleftrightarrow G(\alpha) = 1 - (1-\alpha)^m$$

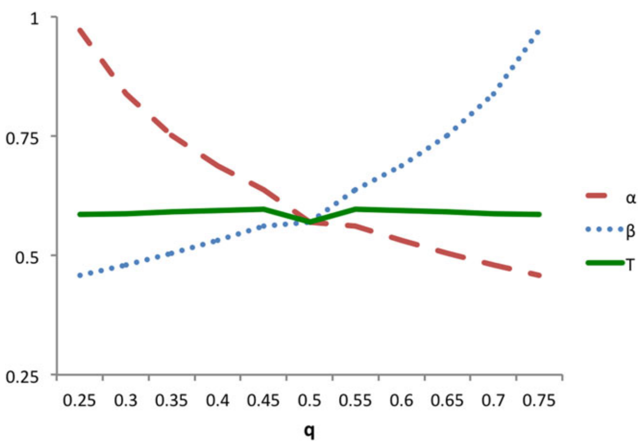
$$c \in [0, 1], \quad m > 0$$

violates the DGRHR property, as the GRHR of the inverse $G(\alpha)$ (see Lemma 1 in the appendix) is decreasing:

$$h(\alpha) = \frac{m\alpha\,(1-\alpha)^{m-1}}{1 - (1-\alpha\alpha)^m}.$$

[15]See, for instance, the papers mentioned in the first paragraph of the introduction.

FIGURE 3 **T, α, and β as a Function of q, Ethical Voter Model**



Take, for instance, $m = 2$. For $\gamma = 1$, the FOCs are

$$q\alpha^2 (2 - \alpha) = (1 - q)\,\beta^2 (2 - \beta) = W,$$

where

$$R = \frac{\beta (2 - \beta)}{\alpha (2 - \alpha)}, \quad W = \frac{\alpha (2 - \alpha)\, \beta (2 - \beta)}{(\alpha (2 - \alpha) + \beta (2 - \beta))^2}$$

and turnout is

$$T = \frac{1}{\alpha (2 - \alpha) + \beta (2 - \beta)}.$$

Figure 3 shows a violation of the competition effect. Namely, despite the presence of the underdog effect, total turnout is not always increasing for $q \in [0, 1/2]$.

Mobilization Model

The main message of this article is to document the robustness of our comparative statics results of turnout across several well-known turnout models. Morton (1987, 1991), Cox and Munger (1989), Shachar and Nalebuff (1999), and others have proposed models based on group mobilization, where parties can mobilize and coordinate citizens to go vote. In major elections, candidates and parties engage in hugely expensive get-out-the-vote drives. Empirical evidence suggests that these drives are effective (Bochel and Denver 1971; Gerber and Green 2000). There is also evidence that mobilization efforts can explain turnout variation across elections and across electoral systems (Gray and Caul 2000; Patterson and Caldeira 1983). We adopt here a group mobilization model á la Shachar and Nalebuff (1999), where parties' campaign efforts and spending are able to mobilize and coordinate citizens to go vote. In this model, each group can "purchase" turnout of its party members by engaging in costly get-out-the-vote efforts. Thus, parties trade off mobilization costs for higher expected vote shares, taking as given the mobilization choice of the other party.

A mobilization model assumes that more campaign spending by a party brings more votes for the party according to an exogenous technology. We consider a very simple version of group mobilization. We assume that the cost a party incurs in order to bring to the polls (i.e., mobilize) all its supporters with voting cost below c is $l(c)$, where $c \in [0, \overline{c}]$ and l is an increasing, convex, and twice differentiable function. We also assume that it is infinitely costly for a party to turn out all its supporters: $l(\overline{c}) = \infty$. In addition to twice differentiability, we assume the distribution of citizens' voting costs $F(c)$ satisfies a (weakly) decreasing reversed hazard rate (DRHR, or log-concavity of F) property.[16]

As in the ethical voter model, to make sure the first order conditions identify actual maximizers we need to impose additional conditions. First, both a large enough cost density for low costs and low enough marginal cost of mobilizing the first voters: this rules out the zero turnout best response, by making it almost costless to turn out the first voters.[17] Second, the log-convexity of the marginal cost of mobilization: this ensures the objectives are single peaked, thanks to the DRHR property. Namely:

Condition 2. *The cdf of the cost distribution $F(c)$ defined on $c \in [0, \overline{c}]$ must satisfy the additional conditions:*

1) $\lim_{c \longrightarrow 0} \frac{l'(c)}{l''(c)} \frac{f(c)}{F(c)} < \frac{1}{\gamma - 1}$
2) *$l(c)$ is such that $l''(c)/l'(c)$ is increasing (log-convexity of $l'(c)$).*

Under the above conditions, we have the following result, without loss of generality, for all $q \in [0, 1/2]$.

Proposition 2. *In the mobilization model, an equilibrium exists, it is unique, and it has the following properties:*

(1) *Zero Underdog Compensation: Regardless of the preference split, both sides turn out the same proportion of supporters, $\alpha = \beta$; hence, $R = Q$ and turnout equals this proportion, $T = \alpha = \beta$.*
(2) *Competition Effect: Given an institutional system γ, turnout increases in the evenness of the preference split, $q < 1/2$, and peaks for an ex ante even election, $q = 1/2$.*

[16]Note that the DRHR (also known as log-concavity of F) is weaker than the DGRHR used in the ethical voter model. Hence, the DGRHR property is sufficient for all the results obtained in both models.

[17]Note that the upper corner solution is already ruled out by $l(\overline{c}) = \infty$.

(3) Contest Effect: Given a preference split q, turnout increases and then decreases as the extent of power sharing drops; that is, as γ increases, it achieves its maximum for intermediate power-sharing systems.

(4) As the system becomes a system with no power sharing, γ goes to infinity, turnout goes to one when the preference split is even, $q = 1/2$, and it goes to zero otherwise.

Proof. See the appendix. ■

The weak DRHR property of the cost distribution roughly means that the relative variation in the number of agent-types does not increase as we span the support of the distribution. In other words, as we increase the cost, we do not suddenly find many more agents with a given cost. This guarantees monotonicity, and it is essential for a unique interior solution: The cost-benefit ratio of turning agents with marginally higher costs is increasing.

The results for this model and their intuition are very similar to the ethical voter model, with one caveat. The zero underdog compensation obtained in the mobilization model means that, regardless of the electoral system and of the preference split, either side turns out the same proportion of its supporters. The zero underdog compensation is due to the nonrival structure of the campaign spending costs in mobilization models (see, e.g., Morton 1987, 1991; Schachar and Nalebuff 1999). Namely, it costs the same for either side to mobilize all their supporters below a given voting cost threshold. In particular, it does not cost less to turn out the same share of supporters of the smaller group than of the larger group. This is, for example, the case if one thinks of campaigning as advertising through media, which is in its nature nonrival, but not for other forms of campaigning such as door-to-door persuasion, which are clearly rival. If the latter were the case, then compensation would be partial and the results would be similar to the ethical voter model.

Example. If the voting cost distribution and the cost of mobilizing voters are:

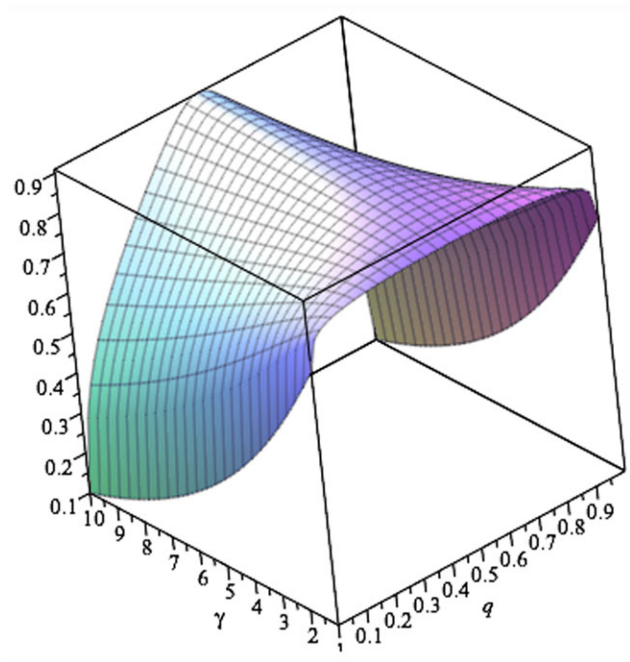
$$F(c) = \left(\frac{c}{a+c}\right)^{1/k}, \quad c \in [0, +\infty), \quad k > \frac{\gamma - 1}{2}$$

$$l'(c) = \frac{c^2}{1+c}$$

then, the FOC becomes:

$$\left(MB := \gamma \frac{Q^\gamma}{[1+Q^\gamma]^2}\right) = \left(\frac{l'(c)}{f(c)/F(c)} = kc^3\right)$$

FIGURE 4 T as a Function of γ and q, Mobilization Model



Hence the solution is:

$$T = \alpha = \beta = F(c^*) = \left(\frac{c^*}{1+c^*}\right)^{1/k},$$

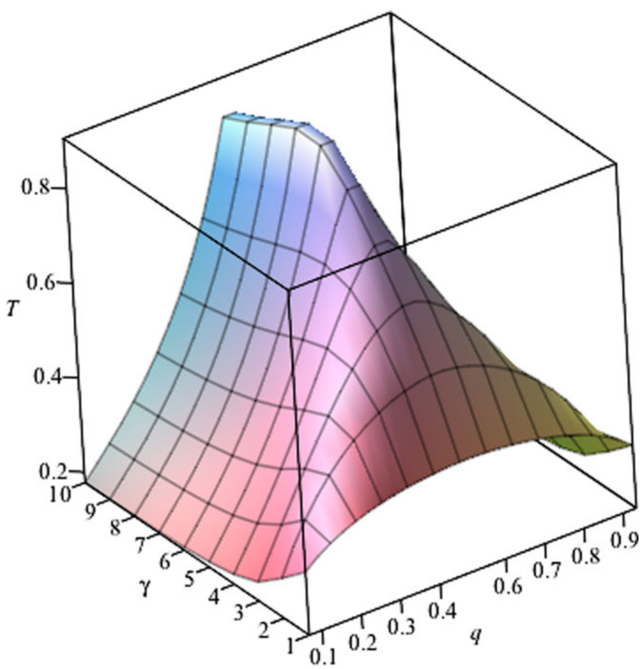
$$\text{with: } c^* = \left(\frac{MB}{k}\right)^{1/3}$$

Figure 4 shows T as a function of both q and γ. The similarity with Figure 2 is apparent. Also, for any electoral/institutional system γ the competition effect is clear, as well as the contest effect for any preference split q.

Rational Voter Model

Another workhorse model for studying turnout in elections is the rational voter model (Palfrey and Rosenthal 1985). Some scholars consider the rational voter model nonsatisfactory, as it predicts very low levels of turnout in large electorates. Far from contributing to this debate, our goal here is rather to show that, regardless of the turnout levels predicted, the comparative statics we obtain in the rational voter model across institutional systems and across preference splits are consistent with what we obtained in the high-turnout-yielding, group models of turnout we discussed above.

FIGURE 5 T as a Function of γ and q, Rational Voter Model



Note: N = 30.

Let N denote the total number of voters. As usual, the symmetric equilibrium is characterized by two cutoff levels, with $\alpha = F(c_\alpha)$ and $\beta = F(c_\beta)$, which solve:

$$MB_A = G(\alpha), \quad MB_B = G(\beta),$$

where MB_A and MB_B are the marginal benefits from voting for an individual of, respectively, group A and group B. Given expected turnout rates in the two parties, α and β, the expected marginal benefits of voting for a party A and party B citizen are equal to, respectively,

$$\sum_{b=0}^{N}\sum_{a=0}^{N-b}[V(a+1,b)-V(a,b)]\frac{N!}{a!b!(N-a-b)!} \times T_A^a T_B^b (1-T_A-T_B)^{N-a-b} \quad (3)$$

$$\sum_{b=0}^{N}\sum_{a=0}^{N-b}[V(b+1,a)-V(b,a)]\frac{N!}{a!b!(N-a-b)!} \times T_A^a T_B^b (1-T_A-T_B)^{N-a-b} \quad (4)$$

where

$$V(a,b) := \frac{a^\gamma}{a^\gamma + b^\gamma}.$$

In Equations (3) and (4), the first term in brackets in the summation is the increase in power share as a consequence of an increase in vote shares. The remaining terms represent the probability of the vote share being equal to $\frac{a}{a+b}$ without your vote, given turnout rates α and β.[18] For this model, we can offer analytical proofs for two results: existence of an equilibrium and the presence of a partial underdog effect. To show that the comparative statics discussed for the previous models hold also under these alternative modeling assumptions, we recur to numerical computations.

Existence. Fix N, γ, and q. The pair of equilibrium conditions can be written in terms of cost thresholds as:

$$MB_A(c_A, c_B) = c_A, \; MB_B(c_A, c_B) = c_B.$$

Because $\overline{c} > 1/2$, MB_A and MB_B are continuous functions of c_A, c_B from $[0, \overline{c}]^2$ into itself, and $[0, \overline{c}]^2$ is a compact convex subset of R^2.[19] Therefore, by Brouwer's theorem, there exists a fixed point (c_A^*, c_B^*) that satisfies both equations and is an equilibrium.

Underdog Effects. This proof is contained in Kartal (Forthcoming a) and Herrera, Morelli, and Palfrey (2014). As pointed out there, in general, the partial underdog effect holds whenever a symmetric power sharing function $V(a, b)$ (e.g., as the purely proportional one with $\gamma = 1$ in our model: $V = \frac{a}{a+b}$) has the property that an additional vote for the underdog has a higher marginal impact for the underdog than an additional vote for the leader has for the leader. In other words, the proof just hinges on following two properties of $W(a, b)$:

$$W(a,b) = -W(b,a), \text{ and } W(a,b) > 0 \text{ if } a < b,$$

where

$$W(a,b) := (V(a+1,b) - V(a,b)) - (V(b+1,a) - V(b,a)).$$

While closed-form analytical expressions of the equilibria do not exist, they are easily computed numerically. Figure 5 shows the equilibrium overall turnout as a function of the institutional environment, γ, and the ex ante preference split, q, for $N = 30$, $c \sim U[0, 1]$, and a benefit from winning the election of 10. This figure is qualitatively similar to the ones we presented for the mobilization and the ethical voter models. Table 1 shows the overall turnout as a function of γ and q for the same parameters. From these numerical computations, we can conclude

[18]By convention, we denote $\frac{j}{j+k} = .5$ if $j = k = 0$.

[19]The assumption $\overline{c} > 1/2$ guarantees that the range of these functions is contained in $[0, \overline{c}]^2$. Existence also holds more generally for any $\overline{c} > 0$, with only minor changes in the proof to account for the possibility that $\overline{c}$ is the cutpoint (i.e., 100% turnout) for one or both parties.

Table 1 T as a Function of γ and q, Rational Voter Model

	γ									
q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0.07	0.311	0.262	0.218	0.199	0.189	0.184	0.181	0.179	0.178	0.178
0.10	0.333	0.318	0.269	0.241	0.223	0.219	0.214	0.211	0.209	0.208
0.13	0.351	0.364	0.318	0.285	0.266	0.255	0.249	0.244	0.241	0.239
0.17	0.365	0.404	0.367	0.331	0.308	0.294	0.285	0.279	0.275	0.272
0.20	0.376	0.437	0.415	0.379	0.352	0.336	0.324	0.316	0.311	0.307
0.23	0.385	0.466	0.462	0.429	0.401	0.381	0.367	0.358	0.351	0.346
0.27	0.393	0.490	0.501	0.482	0.453	0.431	0.415	0.403	0.395	0.389
0.30	0.399	0.510	0.546	0.536	0.511	0.487	0.469	0.455	0.445	0.437
0.33	0.404	0.527	0.583	0.591	0.573	0.550	0.529	0.514	0.502	0.492
0.37	0.408	0.541	0.615	0.644	0.640	0.621	0.600	0.582	0.567	0.556
0.40	0.411	0.551	0.641	0.694	0.711	0.702	0.682	0.662	0.645	0.631
0.43	0.414	0.558	0.660	0.736	0.783	0.795	0.781	0.759	0.738	0.720
0.47	0.415	0.563	0.672	0.765	0.847	0.905	0.906	0.878	0.849	0.823

Note: N $= 30$; $c \sim U[0, 1]$.

that comparative statics similar to the one discussed for the other two models hold.

First, given γ, turnout increases in the size of the underdog group and peaks when the preference split is even. This is an analogue of the competition effect we discussed in the previous two models. Second, turnout increases and then decreases as we approach a system with no power sharing (i.e., as γ grows). In the large majority case, that is, an uneven preference split (e.g., $N_A = 4$ when $N = 30$), turnout is maximized for $\gamma = 1$ and decreases as we increase γ. When preferences are closer (e.g., $N_A = 10$ when $N = 30$), turnout initially increases as we increase γ. As the power sharing of the system becomes less proportional, winning the election becomes paramount, so competition becomes fiercer. On the other hand, the γ that maximizes turnout is still finite (i.e., does not coincide with pure first-past-the-post). As we approach a system without power sharing, the incentive to vote is reduced: Winning becomes all that matters, and the underdog, which has a smaller chance of winning (especially when preferences are uneven) turns out less. This is an analogue of the contest effect we discussed in the previous two models. Finally, we see in both cases the presence of a partial underdog effect. The underdog effect in this model comes from the benefit side, not from the cost side. The underdog has a larger benefit from turning out, as an additional vote for the underdog brings the election closer to a tie, raising the stakes; specifically, the benefit function becomes steeper as we approach a tie. The fact that the underdog effect is partial thought is entirely due to cost heterogeneity, as in the ethical voter model.

Concluding Remarks

This article investigated how the endogenous decisions of voters in participate to an election is affected by the degree of power sharing of the political system. We introduce a novel modeling instrument, a generalized context success function, in order to measure the sensitivity of power sharing to vote shares. Contrary to previous theoretical studies linking turnout to political institutions, this allows us to consider a wide array of both electoral systems—ranging from a perfectly proportional system to a pure winner-take-all system—and also, independently, of power-sharing regimes.

We show that turnout depends on the degree of proportionality of influence in the institutional system in a subtle way, and that it is important to control for the interaction of political institutions and the relative strength of parties in the electorate. With the exception of the knife-edged case of a perfectly even split of preferences, turnout is highest for an intermediate degree of power-sharing. When the distribution of preferences is lopsided, turnout is maximized with a relatively more proportional power-sharing system; when the distribution of preferences is close to even, turnout is maximized with a relatively more uneven power sharing system. The fact that, with low power sharing, the underdog is unlikely to win a large election when partisan preferences are lopsided strongly discourages turnout. On the other hand, with high power sharing, some competition remains even when preferences are uneven, and therefore the effect of relative party support on turnout is small. These theoretical results are robust to a wide range of alternative assumptions about the role of parties and about the rationality of voters.

There are many possible directions for the next steps in this research. In this article, we identify theoretically how the overall proportionality of influence and turnout interact with the distribution of preferences in the elecotrate, and we find that these interaction effects are important, yet quite subtle. A direct empirical test of the theory is not readily available because there are no good measures of the overall degree of power sharing of a political system. There are well-established indices only for the mapping from votes to seats, which is just one component of the broader mapping from votes to power, which is what ultimately matters for the decision of voters to participate in an election. However, as suggested in the appendix, a researcher interested in evaluating our theory can usefully compare how turnout varies with the level of competitiveness when varying only one of the two mappings and keeping the other constant (e.g., varying the degree of power to the legislature while keeping the electoral rule constant, or varying the electoral rule within classes of countries with similarly powerful legislatures). The preliminary tests discussed in the appendix are broadly supportive of our theoretical predictions. In particular, turnout increases significantly more with competitiveness in first-past-the-post (FPTP) systems with respect to proportional representation (PR) systems when keeping one component of the mapping from seats to power constant, and the degree of power conferred to the legislature (which we assume is positively correlated with γ) increases the sensitivity of turnout to competitiveness when focusing exclusively on countries with the same electoral rule. There is a large scope for better empirical tests of this theory, and we believe this offers a fruitful avenue for future research in comparative politics. Finally, on the theoretical side, it would be interesting to study voters' turnout decisions when elections have a common value dimension and how the extent of power sharing in the system affects political platforms and the endogenous entry of parties.

Appendix

The lemma below is used in the proof of Proposition 1, which follows.

Lemma 1. *If a distribution function satisfies the decreasing generalized reversed hazard rate (DGRHR) property, then its inverse satisfies the increasing generalized reversed hazard rate (IGRHR) property, and vice versa.*

Proof of Lemma 1

Omitting the arguments of the functions in our notation, the derivative of the inverse function is well known to be

$$G' = \frac{1}{F'} \quad \Longleftrightarrow \quad g = \frac{1}{f}.$$

Using the chain rule of the above expression, we can obtain the second derivative of the inverse function, that is,

$$G'' = -\frac{F''}{(F')^2}G' = -\frac{F''}{(F')^3} \Longleftrightarrow g' = -\frac{f'}{(f)^3}.$$

The IGRHR property for G, namely,

$$\left(\frac{\alpha g}{G}\right)' > 0 \Longleftrightarrow 1 + \frac{\alpha g'}{g} > \frac{\alpha g}{G}$$

translates, substituting the above expressions, to

$$-\frac{f'}{(f)^3}Ff + 1 > \frac{F}{cf} \quad \Longleftrightarrow \frac{cf}{F} > 1 + \frac{cf'}{f}.$$

which is precisely the DGRHR property for F, and vice versa.

Proof of Proposition 1

Partial Underdog Compensation. Before proving existence and uniqueness (below), we show that any solution must satisfy the partial underdog effect property.

The marginal group benefits to the two parties, with respect to (c_α, c_β), are given by Equation (2). The first-order conditions are

$$\frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\,\beta}{T^2}\right) qf\left(c_\alpha\right) = qc_\alpha f\left(c_\alpha\right) \text{ and}$$

$$\frac{dP_\gamma^A}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\; f\left(c_\beta\right) = (1-q)\,c_\beta f\left(c_\beta\right),$$

which give the condition

$$q\alpha G\left(\alpha\right) = (1-q)\,\beta G\left(\beta\right).$$

If a solution exists, the above condition implies *partial underdog compensation*, namely,

$$q < 1/2 \Longrightarrow a > \beta,\, q\alpha < (1-q)\,\beta.$$

The preference group that is smaller in expectation (i.e., the underdog) turns out a larger fraction of its members than the larger group, that is, $\alpha > \beta$. However, this is not enough to compensate the initial disadvantage in the population, that is, $q\alpha < (1-q)\beta$.

Existence and Uniqueness. We first show that the solution to the system of first order conditions exists and it is unique, then we show that the first order conditions are sufficient conditions. We can write

$$R = \frac{G(\beta)}{G(\alpha)},\ W = \gamma \frac{(G(\alpha))^{\gamma} (G(\beta))^{\gamma}}{((G(\alpha))^{\gamma} + (G(\beta))^{\gamma})^2}$$

and the two FOCs compactly as

$$(1-q)\,\beta G(\beta) = q\alpha G(\alpha) = W.$$

The first equality above implicitly defines the increasing function $\beta(\alpha)$. Taking derivatives, the relative variation of both expressions in the first equality gives:

$$\frac{d\beta}{\beta} + \frac{dG(\beta)}{G(\beta)} = \frac{d\alpha}{\alpha} + \frac{dG(\alpha)}{G(\alpha)}$$

$$\left(1 + \frac{\beta g(\beta)}{G(\beta)}\right)\frac{d\beta}{\beta} = \left(1 + \frac{\alpha g(\alpha)}{G(\alpha)}\right)\frac{d\alpha}{\alpha}$$

Since for $q < 1/2$ we have $\alpha > \beta$, the above implies $\frac{d\beta}{\beta} > \frac{d\alpha}{\alpha}$ under IGRHR. Hence, the function $\frac{G(\alpha)}{G(\beta)}$ is increasing in α, as its relative variation is

$$\frac{d\left(\frac{G(\alpha)}{G(\beta)}\right)}{\frac{G(\alpha)}{G(\beta)}} = \frac{dG(\alpha)}{G(\alpha)} - \frac{dG(\beta)}{G(\beta)} = \frac{d\beta}{\beta} - \frac{d\alpha}{\alpha} > 0.$$

The second equality in the FOCs above can be written as

$$q\alpha G(\alpha) = \gamma \frac{\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}}{\left(\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma} + 1\right)^2}.$$

To show uniqueness, note that the left-hand side (LHS) is strictly increasing in α, whereas the right-hand side (RHS) is strictly decreasing because for all $\gamma \geq 1$,

$$\frac{d}{dx}\left(\gamma \frac{x^{\gamma}}{(x^{\gamma}+1)^2}\right) < 0 \text{ for } x > 1.$$

To show existence, note that for $\alpha = 0$ the LHS is zero while the RHS is bounded by 1 as

$$\frac{G(\alpha)}{G(\beta)} > 1 \Longrightarrow \lim_{\alpha \to 0} \frac{G(\alpha)}{G(\beta)} \geq 1.$$

On the other hand, for $\alpha = 1$, the LHS is equal to q while, given that $\frac{G(\alpha)}{G(\beta)} = \frac{(1-q)\beta}{q}$, the RHS is bounded by

$$\gamma \frac{\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma}}{\left(\left(\frac{G(\alpha)}{G(\beta)}\right)^{\gamma} + 1\right)^2} = \gamma \frac{q^{\gamma}}{\left(1 + \left(\frac{q}{(1-q)\beta}\right)^{\gamma}\right)^2}$$

$$< \gamma \frac{q^{\gamma}}{\left(1 + \left(\frac{q}{(1-q)}\right)^{\gamma}\right)^2} \leq q,$$

where the latter inequality is true for all $\gamma \geq 1$.

We now show that the corner colutions are ruled out and the objectives are single peaked, hence the first order conditions identify the global maximum, under the boundary conditions on $F(c)$.

The objective function of, wlog, A (renaming A's choice variable: $c_{\alpha} = c$) is:

$$U(c) := P_{\gamma}^{A} - C(c)$$

Hence its derivative decomposes into a product:

$$U'(c) = f(c)\left(\frac{dP_{\gamma}^{A}}{dV}\frac{q(1-q)\beta}{T^2} - qc\right) = \Psi(c)\Omega(c)$$

in which the two factors are:

$$\Psi(c) := q\Delta \frac{f(c)}{F(c)}, \quad \Omega(c) := \left(\frac{\gamma}{q} - c\frac{F(c)}{\Delta}\right)$$

and:

$$\Delta := \frac{hF^{\gamma}(c)}{(1 + hF^{\gamma}(c))^2}, \quad h := \left(\frac{q}{1-q}\frac{1}{\beta}\right)^{\gamma} \in (0, +\infty)$$

namely h includes also the strategy β of player B (taken as given).

To have interior single-peakedness of $U(c)$ at some interior $c^* \in (0, \bar{c})$, it suffices to have:

$$U' > 0 \quad \text{for} \quad 0 < c < c^* \quad \text{and} \quad U' < 0 \quad \text{for} \quad c > c^*$$

For this purpose, since for all $c > 0$ the factor $\Psi(c)$ is positive, it suffices that the factor $\Omega(c)$ is:

1) eventually negative (i.e., at $c = \bar{c}$ thus by continuity in a neighborhood of $\bar{c}$);
2) initially positive (namely, at $c = 0$ thus by continuity in a neighborhood of 0);
3) strictly decreasing (namely, has a unique zero which thus coincides with c^*).

To satisfy the three conditions above the key magnitude to study is:

$$c\left(\frac{1}{hF^{\gamma-1}(c)} + 2F(c) + hF^{\gamma+1}(c)\right) > \frac{\gamma}{q} \iff \Omega(c) < 0$$

1) Recalling that $F(\bar{c}) = 1$, a sufficient condition for $\Omega(\bar{c}) < 0$ is $\bar{c}$ large enough:
$$\bar{c} > \frac{\gamma}{4q} \Rightarrow \bar{c}\left(\frac{1}{h} + 2 + h\right) > \frac{\gamma}{q}$$
2) Recalling that $F(0) = 0$, a sufficient condition for $\Omega(0) > 0$ is:
$$\lim_{c \longrightarrow 0} \frac{c}{F^{\gamma-1}(c)} = 0$$
3) Recalling that $F(c)$ is (weakly) increasing, a sufficient condition for single peakedness is for the term $\frac{c}{F^{\gamma-1}(c)}$ to be increasing, which, inspecting the derivative, is implied by:
$$\frac{cf(c)}{F(c)} < \frac{1}{\gamma - 1}$$

The DGRHR property ensures the LHS is decreasing, thus it suffices to have:

$$\lim_{c \longrightarrow 0} \frac{cf(c)}{F(c)} < \frac{1}{\gamma - 1}$$

In sum, both limit expressions above are satisfied for any cdf $F(c)$ which is asymptotically larger than or equal to $c^{1/k}$ (in a neighborhood of zero), namely:

$$\lim_{c \longrightarrow 0} \frac{F(c)}{c^{1/k}} > 0 \quad \text{for } k > \gamma - 1$$

Competition Effect. Take wlog $q < 1/2$; hence, $\alpha > \beta$ and $R < 1$. We now show that under IGRHR, if q increases then both R and T increase in the equilibrium solution.

Fixing γ, W increases with R as for the partial derivative of W we have:

$$W_R = R^{\gamma-1}\gamma^2 \frac{1 - R^\gamma}{(R^\gamma + 1)^3} > 0 \text{ for } R < 1.$$

Suppose, by contradiction, that, in equilibrium, if q increases R decreases. Then $W = q\alpha G(\alpha) = (1-q)\beta G(\beta)$ decreases, which implies that α decreases. Hence, β decreases too as $R = \frac{G(\beta)}{G(\alpha)} = \frac{q\alpha}{(1-q)\beta}$, which in turn implies that $\frac{\alpha}{\beta}$ is decreasing and $\frac{G(\alpha)}{G(\beta)}$ increasing, a contradiction under IGRHR. In fact, the IGRHR property guarantees, for $\alpha > \beta$, that:

$$d\left(\frac{\alpha}{\beta}\right) > 0 \Longleftrightarrow \frac{d\alpha}{\alpha} > \frac{d\beta}{\beta}$$

$$d\left(\frac{G(\alpha)}{G(\beta)}\right) > 0 \Longleftrightarrow h(\alpha)\left(\frac{d\alpha}{\alpha}\right) > h(\beta)\left(\frac{d\beta}{\beta}\right),$$

which implies the two ratios (i.e., the turnout ratio α/β and the cost threshold ratio c_α/c_β) must move in the same direction, namely,

$$d\left(\frac{\alpha}{\beta}\right) > 0 \Longrightarrow d\left(\frac{G(\alpha)}{G(\beta)}\right) > 0$$

$$d\left(\frac{G(\alpha)}{G(\beta)}\right) < 0 \Longrightarrow d\left(\frac{\alpha}{\beta}\right) < 0$$

We have shown that R cannot decrease when q increases. As a consequence, $W = q\alpha G(\alpha) = (1-q)\beta G(\beta)$ increases (which means that β increases, that is, the front-runner group turns out more when the ex ante lead shrinks). In formulas, this implies that:

$$0 < \frac{d\left(q\alpha G(\alpha)\right)}{dq} = \alpha G(\alpha) + q\left(G(\alpha) + \alpha g(\alpha)\right)\alpha_q$$

$$\Longleftrightarrow q\alpha_q > -\frac{\alpha}{1 + h(\alpha)}$$

and

$$0 < \frac{d\left((1-q)\beta G(\beta)\right)}{dq} = -\beta G(\beta) + (1-q)\left(G(\beta) + \beta g(\beta)\right)\beta_q \Longleftrightarrow (1-q)\beta_q > \frac{\beta}{1 + h(\beta)}.$$

Hence, the variation of turnout with competition is:

$$T_q = \left(\alpha + q\alpha_q - \beta + (1-q)\beta_q\right) > \alpha\left(1 - \frac{1}{1 + h(\alpha)}\right) - \beta\left(1 - \frac{1}{1 + h(\beta)}\right).$$

So turnout is increasing if the function h is increasing. In sum under IGRHR for $q \in [0, 1/2]$ we have:

$$T_q > 0, \quad R_q > 0.$$

Contest Effect. Fixing the preference split $q < 1/2$, we study turnout as the contest becomes more competitive, namely, increasing γ from 1 (proportional power sharing) to infinity (no power sharing).

Taking the total derivative with respect to γ of the first-order conditions, we have

$$\begin{aligned} W_\gamma &= q\left(G(\alpha) + \alpha g(\alpha)\right)\alpha_\gamma \\ &= (1-q)\left(G(\beta) + \beta g(\beta)\right)\beta_\gamma. \end{aligned}$$

This implies that W_γ, α_γ, and β_γ and hence also the variation of turnout

$$T_\gamma = q\alpha_\gamma + (1-q)\beta_\gamma$$

always have the same sign and (if applicable) are maximized for the same value $\widehat{\gamma}(q)$. In sum, T, α, and β are increasing in γ if and only if $W_\gamma > 0$, so it suffices to study when the latter is the case. Defining

$$z := R^\gamma \in [0, 1]$$

$$z_\gamma = (\ln R)\, R^\gamma R_\gamma = (-\ln R)\, R^\gamma \left(R\left(\frac{\beta_\gamma}{\beta} - \frac{\alpha_\gamma}{\alpha}\right)\right).$$

Taking the total derivative of W with respect to γ, we have

$$\begin{aligned} W_\gamma &= z\frac{(\ln z)(1-z) + 1 + z}{(1+z)^3} z_\gamma \\ &= \left(z^2 \frac{(\ln z)(1-z) + 1 + z}{(1+z)^3}\right) \\ &\quad \times \left((-\ln R)\, R\left(\frac{\beta_\gamma}{\beta} - \frac{\alpha_\gamma}{\alpha}\right)\right). \end{aligned}$$

The second bracket on the RHS is positive because $R < 1$, and under the IGRHR we have $\beta_\gamma/\beta > \alpha_\gamma/\alpha$. Namely, manipulating the total derivative of the first-order conditions, we obtain

$$q\left(\alpha G(\alpha)+\alpha^2 g(\alpha)\right)\frac{\alpha_\gamma}{\alpha}=(1-q)\left(\beta G(\beta)+\beta^2 g(\beta)\right)\frac{\beta_\gamma}{\beta}$$

$$\frac{\beta_\gamma}{\beta}\Big/\frac{\alpha_\gamma}{\alpha}=\frac{q\alpha G(\alpha)}{(1-q)\beta G(\beta)}\frac{1+\frac{\alpha g(\alpha)}{G(\alpha)}}{1+\frac{\beta g(\beta)}{G(\beta)}}$$

$$=\frac{1+\frac{\alpha g(\alpha)}{G(\alpha)}}{1+\frac{\beta g(\beta)}{G(\beta)}}>1.$$

Hence, the sign of W_γ depends on the first factor:

$$W_\gamma>0\iff\left(z^2\frac{(\ln z)(1-z)+1+z}{(1+z)^3}=0\right)>0$$
$$\iff z>z*$$

Hence it suffices to study when the function z is above the threshold z^*. Namely, turnout is increasing for all values (q,γ) for which

$$z=R^\gamma=\left(\frac{q\alpha}{(1-q)\beta}\right)^\gamma>z^*.$$

We can equivalently define the function

$$Z(\gamma,q):=\ln R^\gamma=-\gamma\left(\ln\frac{(1-q)\beta}{q\alpha}\right)$$

and explore when

$$Z(\gamma,q)>\ln z^*\simeq-1.5.$$

The function Z has the following properties:

$$Z(1,0)=-\infty<\ln z^*<-1=Z(1,1/2),$$
$$Z(\infty,q)=-\infty<\ln z^*$$

$$Z_\gamma(\gamma,q)\;=-\ln\frac{(1-q)\beta}{q\alpha}-\gamma\left(\frac{\beta_\gamma}{\beta}-\frac{\alpha_\gamma}{\alpha}\right)<0,$$
$$Z_q(\gamma,q)>0.$$

Namely, Z decreases in $\gamma\in[1,+\infty)$ for all $q\in[0,1/2]$ and, given γ, increases in q (as $R_q>0$). Hence, for low enough $q\in(0,1/2)$, we have $Z<\ln z^*$ and turnout decreases for all $\gamma\in[1,+\infty)$. As q increases, eventually we have $Z<\ln z^*$ and turnout increases in γ, is highest for intermediate system $\widehat{\gamma}(q)$, and then drops as $Z(\infty,q)<\ln z^*$. Lastly, for $q=1/2$, we have $z=1>z^*$, so turnout is always increasing in γ.

First-Past-the-Post Case. For $q=1/2$, as $\gamma\to+\infty$, we have $\alpha G(\alpha)=\beta G(\beta)=2W=\gamma/2$, which gives the corner solution: $\alpha=\beta=1$. For $q<1/2$, as $\gamma\to+\infty$, given that $R<1$ (partial underdog effect), we have $W=\alpha=\beta=0$. Hence, in first-past-the-post, we have full turnout in an evenly split election and zero turnout otherwise.

Proof of Proposition 2

Existence and Uniqueness. We first show that the solution to the system of first order conditions exists and it is unique.

Given Equation (2), the first-order conditions that characterize the solution are

$$\frac{dP_\gamma^A}{dV}\left(\frac{(1-q)\beta}{T^2}\right)qf(c_\alpha)=l'(c_\alpha),$$
$$\frac{dP_\gamma^A}{dV}\left(\frac{q\alpha}{T^2}\right)(1-q)\,f\left(c_\beta\right)=l'\left(c_\beta\right).$$

Taking the ratio of the two, we obtain

$$\frac{l'(c_\alpha)}{f(c_\alpha)/F(c_\alpha)}=\frac{l'\left(c_\beta\right)}{f\left(c_\beta\right)/F(c_\alpha)}.$$

Hence, assuming that F satisfies DRHR (decreasing reversed hazard rate, or the log-concavity of F), together with the convexity of $l(c)$, is sufficient to obtain the following *zero underdog compensation* condition:

$$c_\alpha=c_\beta\Longrightarrow\alpha=\beta.=T.$$

That is, both parties turn out an identical proportion of their supporters regardless of the preference split q.

The mobilization model reduces therefore to one equation in one unknown, equating marginal benefit (MB) and marginal cost (MC):

$$\left(MB:=\gamma\frac{Q^\gamma}{[1+Q^\gamma]^2}\right)=\left(\frac{l'(c_\alpha)}{f(c_\alpha)/F(c_\alpha)}:=MC\right).\tag{5}$$

The solution $c_\alpha\in[0,\overline{c}]$ exists and is unique because $MC(c_\alpha)$ is increasing, $l(\overline{c})=\infty$, and l is convex. Therefore, $l'(\overline{c})=MC(\overline{c})=\infty$, and $MC(0)=F(0)=0$.

The proof that the first order conditions identify the global maximum (hence that corner solutions are ruled out and the objectives are single peaked) is essentially identical to the one presented for the ethical voter model (replacing qc with $l'(c)$).

Competition Effect. For any fixed cost distribution and cost mobilization function, turnout depends only on the marginal benefit (MB) and increases with it. Hence, in what follows, we study MB as a proxy for turnout T. Fixing the institutional setting γ, we study turnout as we increase the ex ante preference split q from zero (landslide) to 1/2 (close election). We have $\frac{dMB}{dq}=\frac{1}{(1-q)^2}\frac{dMB}{dQ}$; hence, we can focus only on the sign of the derivative with respect to Q:

$$\frac{dMB}{dQ}=\frac{d}{dQ}\frac{\gamma Q^\gamma}{[1+Q^\gamma]^2}=\gamma^2Q^{\gamma-1}\frac{1-Q^\gamma}{(1+Q^\gamma)^3}>0.$$

Hence, regardless of the institutional setting, as the preference split becomes tighter the marginal benefit of voting (MB) and turnout T increase.

Contest Effect. Fixing the preference split q, we study turnout as the contest becomes more competitive, namely, if we increase γ from 1 (proportional power sharing) to infinity (no power sharing). We have

$$\frac{dMB}{d\gamma} = \frac{d}{d\gamma}\frac{\gamma Q^{\gamma}}{[1+Q^{\gamma}]^2} = \left(\frac{Q^{\gamma}(1-Q^{\gamma})}{(1+Q^{\gamma})^3}\right) \times \left(\frac{1+Q^{\gamma}}{1-Q^{\gamma}} + \ln Q^{\gamma}\right) > 0.$$

For any $Q < 1$, the first factor is always positive and does not change the sign of the slope of MB or its maximum. Therefore, the above condition is equivalent to

$$\Omega := \left(\frac{1+Q^{\gamma}}{1-Q^{\gamma}} + \ln Q^{\gamma}\right) > 0,$$

where the function Ω is decreasing in γ, as

$$\Omega_{\gamma} = \ln Q \frac{Q^{2\gamma}+1}{(Q^{\gamma}-1)^2} < 0.$$

Hence, Ω can cross zero only once, where turnout is highest, that is, for

$$\widehat{\gamma}(q) : \Omega(\widehat{\gamma}) = 0.$$

In sum, for any given preference split $q < 1$, turnout is highest for an intermediate system $\widehat{\gamma}(q)$:

$$\widehat{\gamma}_Q = -\left(\frac{\partial \Omega}{\partial Q}\right) / \left(\frac{\partial \Omega}{\partial \gamma}\right) > 0,$$

where

$$\Omega_Q = \frac{\gamma}{Q}\frac{1+Q^{2\gamma}}{(1-Q^{\gamma})^2} > 0.$$

First-Past-the-Post Case. Lastly, it is easy to see that

$$\lim_{\gamma\to\infty} MB = \lim_{\gamma\to\infty}\left(\gamma \frac{Q^{\gamma}}{[1+Q^{\gamma}]^2}\right) = \begin{cases} +\infty & \text{if } Q = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Hence, in first-past-the-post, we have full turnout in an evenly split election and zero turnout otherwise.

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